Quantum Proofs of Proximity

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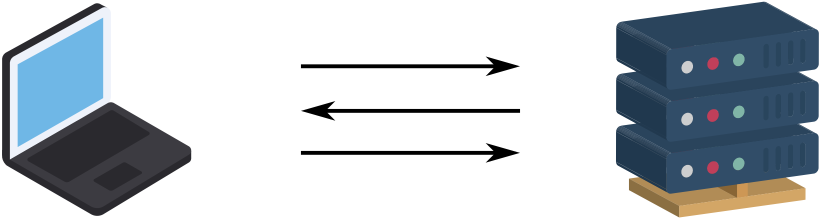
Introduction

Massive datasets, IoT: devices with *dramatically* different power.



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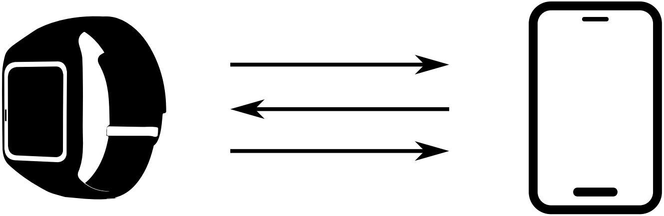

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$$V \quad \begin{array}{c} \xrightarrow{\hspace{3em}} \\ \xleftarrow{\hspace{3em}} \\ \xrightarrow{\hspace{3em}} \end{array} \quad P$$

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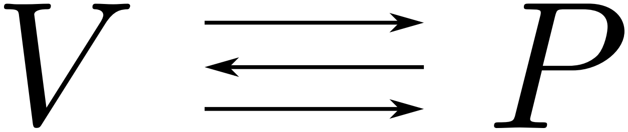

Delegation of computation: *prover* computes, *verifier* checks.

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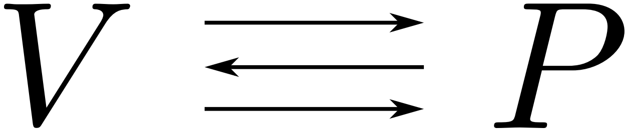
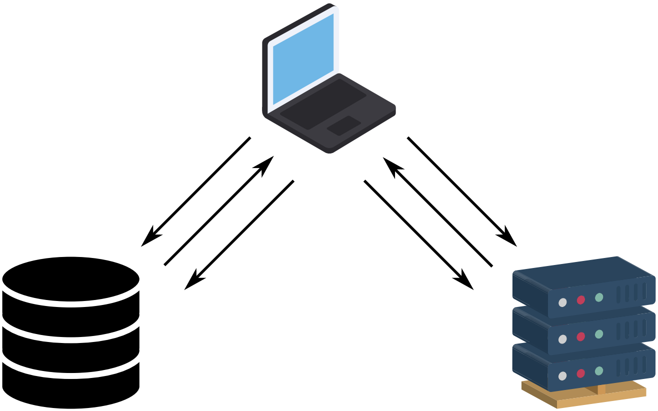
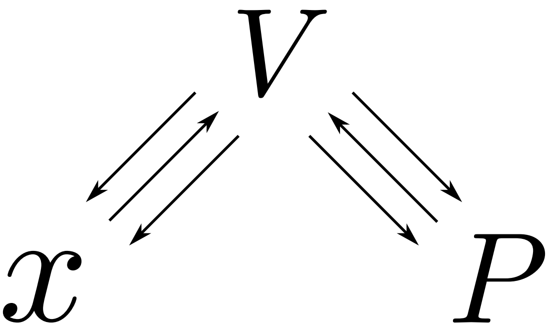


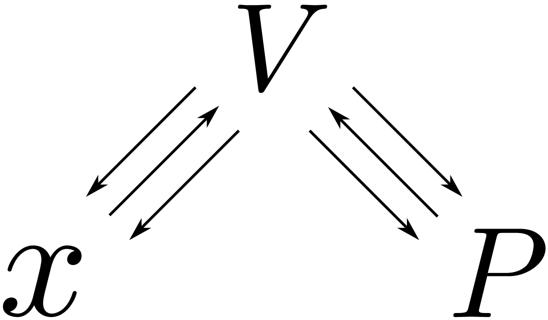
Delegation of computation: *prover* computes, *verifier* checks.
Efficient: $\tilde{O}(n)$ verifier runtime.

Proofs. . .





V checks if $x \in L$ by *querying* x and *communicating* with P.



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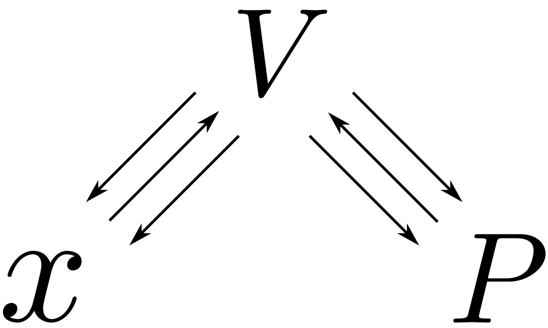
Query complexity q, communication complexity c.

Proofs...



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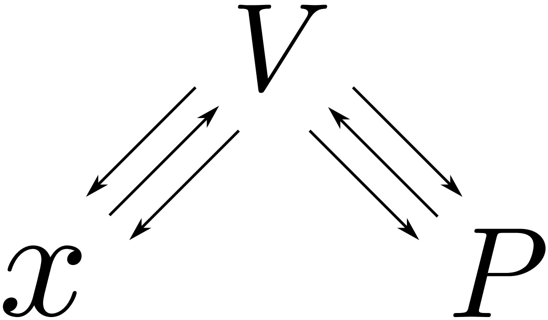
Efficient: $o(n)$ queries and communication.

Proofs...

Example: Does x contain a 1?



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$$\begin{tikzcd} & V \arrow[dl, shift left] & \\ x \arrow[ur, shift left] & & P \arrow[ul, "i"'] \end{tikzcd}$$

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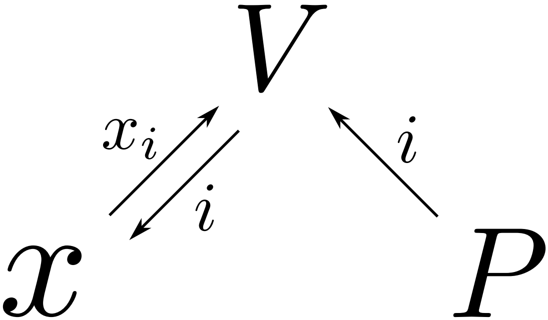

Query complexity 1, communication complexity $\log n$.

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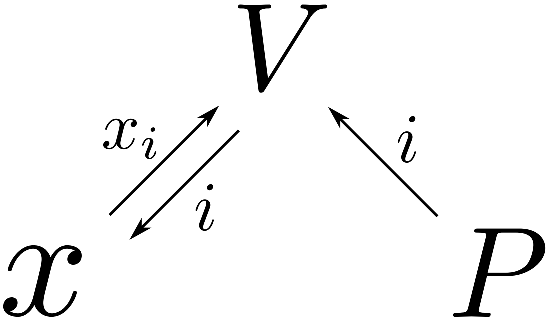
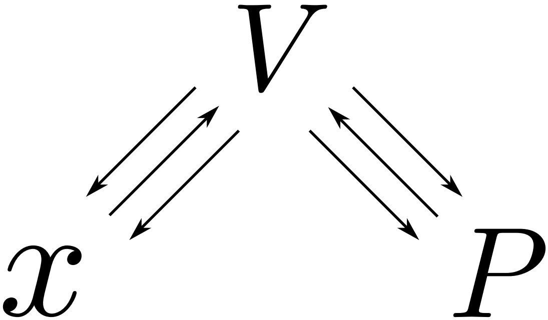
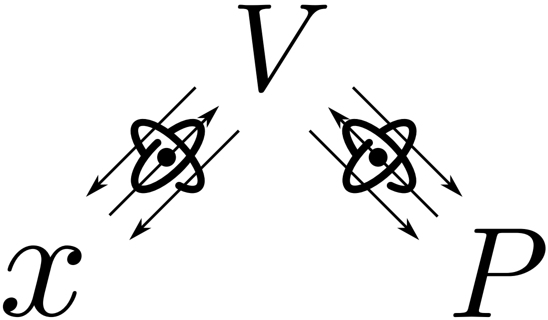


Query complexity 1, communication complexity $\log n$.
($\Omega(n)$ with no proof!)

. . . of proximity. . .



V checks if $x \in \Pi$ **or x is far from** Π (property testing).
Query complexity q, communication complexity c.



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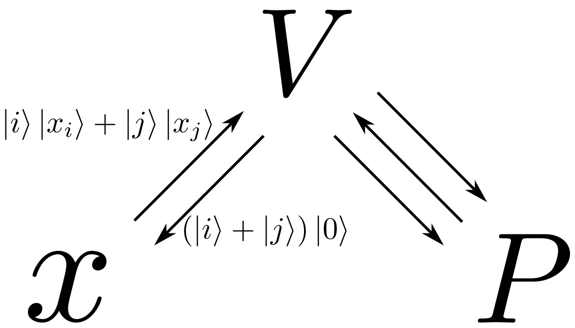
V
$|i\rangle|x_i\rangle + |j\rangle|x_j\rangle$
$(|i\rangle + |j\rangle)|0\rangle$
x
P

Complexity classes

	V	$V \leftarrow P$	$V \leftrightarrow P$
Classical	$\mathcal{PT}$	$\mathcal{MAP}$	$\mathcal{IPP}$
Quantum	$\mathcal{QPT}$	$\mathcal{QMAP}$	$\mathcal{QIPP}$

Also $\mathcal{QCMAP}$: classical proof, quantum input access.

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Also $\mathcal{QCMAP}$: classical proof, quantum input access.

$$\mathcal{C} := \mathcal{C}(\varepsilon, c, q) \text{ with } c, q = \mathrm{polylog}(n) \text{ and}$$
$$\varepsilon \text{ a small enough constant.}$$

Main result

Theorem

The following separations hold:

- *Quantum input access with a proof are more powerful in tandem than separately, i.e.,* $\mathcal{QMAP} \not\subseteq \mathcal{MAP} \cup \mathcal{QPT}$;
- *Classical proofs are weaker than quantum even with a quantum verifier, i.e.,* $\mathcal{QMAP} \not\subseteq \mathcal{QCMAP}$;
- *Quantum proofs cannot substitute for interaction, i.e.,* $\mathcal{IPP} \not\subseteq \mathcal{QMAP}$.

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(Also, some inclusions in the polynomial-time setting carry over)

Main result

$$
\begin{tikzcd}
 & \mathcal{QIPP} & \\
\mathcal{IPP} \arrow[ur] \arrow[rr, "/" marking] & & \mathcal{QMAP} \arrow[ul] \arrow[d, shift left, "/" marking] \\
 & & \mathcal{QCMAP} \arrow[u, shift left] \\
\mathcal{MAP} \arrow[uu] \arrow[urr] \arrow[rr, shift left, "/" marking] & & \mathcal{QPT} \arrow[u] \arrow[ll, shift left, "/" marking] \\
 & \mathcal{PT} \arrow[ul] \arrow[ur] &
\end{tikzcd}
$$

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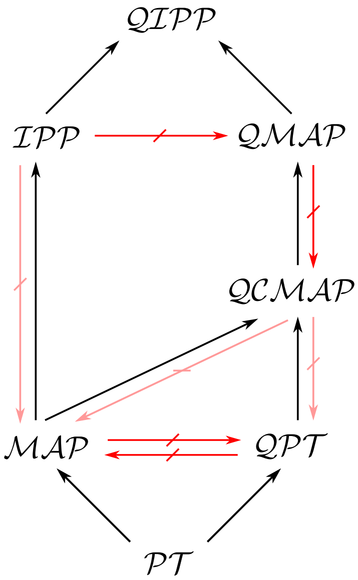

Techniques

$\mathcal{MAP} \not\subseteq \mathcal{QPT}$: disjointness + relaxed locally decodable code
$\mathcal{QPT} \not\subseteq \mathcal{MAP}$: Forrelation
$\mathcal{QMAP} \not\subseteq \mathcal{QCMAP}$: recasting $\mathcal{QMA} \not\subseteq \mathcal{QCMA}$ [AK07]
$\mathcal{IPP} \not\subseteq \mathcal{QMAP}$: permutation testing

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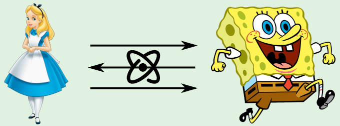

$x \in \{0,1\}^n$ $y \in \{0,1\}^n$

Given $C(x)$ and $C(y)$,
$\exists i \in [n]$ such that $x_i = y_i = 1$?

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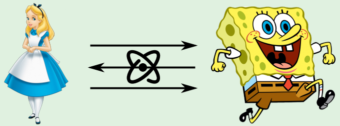

$x \in \{0,1\}^n$ $y \in \{0,1\}^n$

Given $C(x)$ and $C(y)$,
$\exists i \in [n]$ such that $x_i = y_i = 1$?

- $\Omega(\sqrt{n})$ without proof
- $O(1)$ with $\log n$ proof

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$\mathcal{MAP} \not\subseteq \mathcal{QPT}$: disjointness + relaxed locally decodable code
$\mathcal{QPT} \not\subseteq \mathcal{MAP}$: Forrelation

Given $f, g : \{0,1\}^{\log n} \to \{0,1\}$, is $\langle f, \hat{g} \rangle$ small?

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Given $f, g : \{0,1\}^{\log n} \to \{0,1\}$, is $\langle f, \hat{g} \rangle$ small?

- $O(1)$ without proof
- $c \cdot q = \Omega(n^{1/4})$ with proof

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Given $f : [n] \to [n]$, is f a permutation?

- $O(1)$ with (classical) interaction
- $c \cdot q = \Omega(n^{1/3})$ with (non-interactive) quantum proof

Next goal

Theorem ([RVW13], [RR20])

Every language in logspace-uniform NC admits a doubly-efficient IPP with communication and query complexities $\tilde{O}(\sqrt{n})$.

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Every language in logspace-uniform NC admits a doubly-efficient IPP with communication and query complexities $\tilde{O}(\sqrt{n})$. (Prover runtime $\mathrm{poly}(n)$, verifier runtime $\tilde{O}(\sqrt{n})$.)

Theorem (Hopefully!)

QIPPs with complexities $O(n^{\alpha})$ for some $\alpha < 1/2$.

References

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