Quantum Proofs of Proximity

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Introduction

Part I: Quantum algorithms

Part II: Complexity separations

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Introduction: $\mathcal{QMA}$

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Decide language $\mathcal{L}$ in polynomial time, with non-interactive proof.

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Delegation of computation: *prover* computes, *verifier* checks.

$$P \xrightarrow[|\psi\rangle]{} V(x)$$

Given $x \in \{0,1\}^n$ and a poly(n)-qubit state $|\psi\rangle$,

- if $x \in \mathcal{L}$, $\exists\, |\psi\rangle$ such that V accepts w.p. $\geq 2/3$;
- if $x \notin \mathcal{L}$, $\forall\, |\psi\rangle$, V accepts w.p. $\leq 1/3$.

V runs in poly(n) time. [Kitaev et al., 2002]

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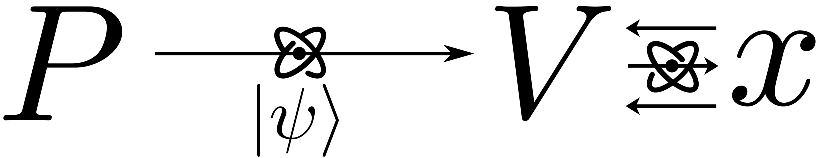
V runs in $o(n)$ time?

$\mathcal{QMA}$ + property testing



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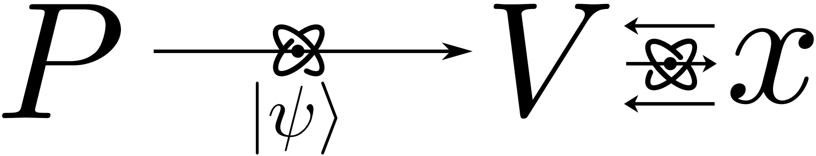

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$\mathcal{QMAP}(\varepsilon, p, q)$:

properties Π such that…

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Theorem (Amplitude amplification [Brassard et al., 2002])

If a one-sided randomised algorithm makes q queries and detects an error with probability ρ, there is a quantum algorithm making $O(q/\sqrt{\rho})$ queries that succeeds w. p. $2/3$.

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Quantumly, an $n^{2/3}$-bit proof and $O(n^{2/3})$ queries suffice!

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$x =$ 

If the parity of the proof π is odd, reject.
Sample $i \in [p]$ uniformly and query the i^{th} block of n/p bits. Accept if their parity matches π_i, reject otherwise.

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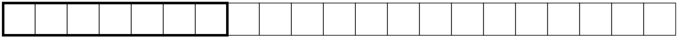
$x =$ 

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$$\Downarrow$$

Seting $p = n^{2/3}$ in the previous algorithm, it makes $n^{1/3}$ queries to detect an error with probability $1/n^{2/3}$. Therefore,

$$q = O\left(\frac{n^{1/3}}{\sqrt{1/n^{2/3}}}\right) = O(n^{2/3}).$$

Theorem

A similar strategy works for every decomposable property.

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Includes:

- k-monotonicity;
- acceptance by branching programs;
- membership in context-free languages;
- Eulerian graph orientations.

Techniques

Decomposable properties: known *classical* proofs of proximity
[Gur and Rothblum, 2018, Goldreich et al., 2018]

Bipartiteness: Quantum collision-finding algorithm
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Complexity classes

	V	$V \leftarrow P$	$V \leftrightarrow P$
Classical	$\mathcal{P}$	$\mathcal{NP}$	$\mathcal{IP}$
Quantum			

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- $\mathcal{IPP} \not\subseteq \mathcal{QMAP}$, *i.e., quantum proofs cannot substitute for interaction.*

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$$\begin{tikzcd}
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$\mathcal{PT}$ lower bounds via communication complexity have proven very successful. [Blais et al., 2012]

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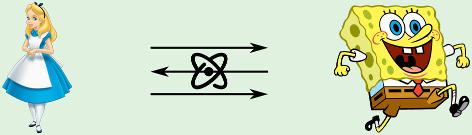

$x \in \{0,1\}^n$ $y \in \{0,1\}^n$

Is there $i \in [n]$ such that $x_i = y_i = 1$?

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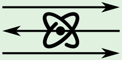

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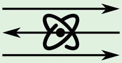

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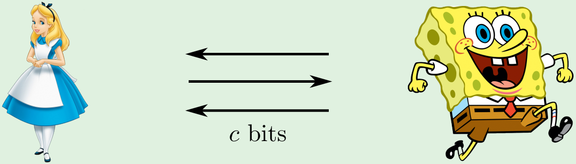

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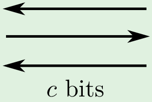
A: b_2.



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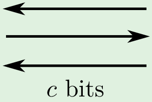

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Solving disjointness with $c \cdot q = \Omega(n)$ bits of communication

$$\Downarrow$$

$$q = \Omega(n/c)$$

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$B \coloneqq \{\, C(x) : x \in \{0,1\}^n \,\}$ are the encodings of Boolean messages.

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$$\Downarrow$$
Quantum ε-tester for B can be used to solve disjointness!

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Goal: simulate a query $|i\rangle |z\rangle \mapsto |i\rangle |z + C(x+y)_i\rangle$

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$\left(\sum_i |i\rangle\right)|z\rangle$

$\mathcal{MAP} \not\subseteq \mathcal{QPT}$: disjointness + relaxed LDC

Let $C : \mathbb{F}^n \mapsto \mathbb{F}^N$ be a *linear code* with distance ε.
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Each query is simulated by $O(\log N)$ qubits of communication.

$\mathcal{MAP} \not\subseteq \mathcal{QPT}$: disjointness + relaxed LDC

Quantum ε-tester for B with q queries

$\Downarrow$

Protocol with $O(q \log N)$ communication complexity

C locally testable and relaxed locally decodable with $N = n^{1.001}$, [Ben-Sasson et al., 2006]

- $C \setminus B \notin \mathcal{QPT}(\varepsilon, n^{0.49})$

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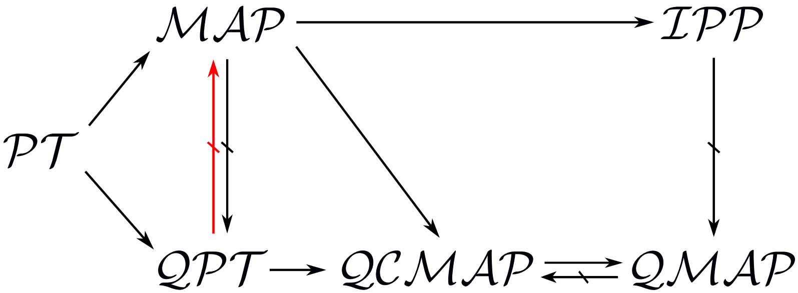
- $C \setminus B \notin \mathcal{QPT}(\varepsilon, n^{0.49})$
- $C \setminus B \in \mathcal{MAP}(\varepsilon, \log n, O(1))$

(Proof points to non-Boolean $i \in [n]$; verifier tests membership in C then decodes i^{th} coordinate and checks if it is Boolean.)

Other separations

$\mathcal{QPT} \not\subseteq \mathcal{MAP}$: Forrelation [Aaronson and Ambainis, 2018]



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$\mathcal{QPT} \not\subseteq \mathcal{MAP}$: Forrelation [Aaronson and Ambainis, 2018]
$\mathcal{QMAP} \not\subseteq \mathcal{QCMAP}$: recasting $\mathcal{QMA} \not\subseteq \mathcal{QCMA}$
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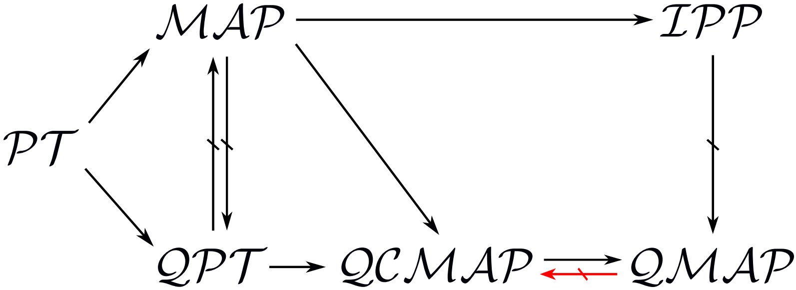

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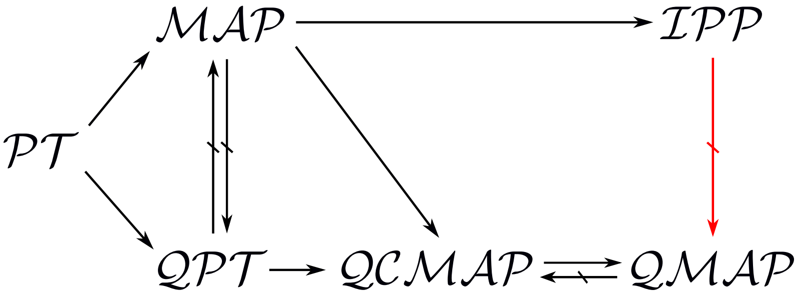
$\mathcal{IPP} \not\subseteq \mathcal{QMAP}$: permutation testing [Gur et al., 2018, Sherstov and Thaler, 2019]



Open problems

- What about $\mathcal{QIPP}$?

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Thank you!

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References III

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